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ASYMPTOTIC MODEL OF ACTIVE RESONANCE ABSORBER OF ACOUSTIC VIBRATIONS IN A CLOSED REGION

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Resonance absorbers of the Helmholtz resonator type are widely used in engineering to suppress acoustic vibrations. The Helmholtz resonator in its classical form operates as a reactive damper and has the drawback that the frequency range in which it works efficiently is limited. It was found that this drawback can be considerably rectified by forming a jet issuing from the throat of a resonator. In this case, a part of the acoustic energy is spent in the generation of an unsteady vortex sheet shedding from the edge of the resonator throat, and thus active absorption of acoustic vibrations occurs.

Some laws governing the influence of the jet on the utilization efficiency of the resonator as a dynamic absorber of acoustic vibrations were studied numerically in the context of a two-dimensional model [1]. In the present paper the question is considered within the framework of a more general spatial statement of the problem. Analytic dependences of the amplitude of forced acoustic vibrations in a closed region on the parameters of the active resonator are obtained in the asymptotic approximation.

1. Basic Assumptions and Small Parameters of the Problem. Let us consider forced acoustic vibrations in a closed region D_0. To suppress the vibrations the Helmholtz resonator, which is an integration of the regions D_1 and D_2, is appended to the region (Fig. 1). Prescribing the form of the resonator throat D_1 as a circular cylinder, let us take as characteristic dimensions of the region $D = D_0 \cup D_1 \cup D_2$ the radius R_1 and length l of the cylinder, and the radii R_0 and R_2 of the spheres, whose volumes V_0 and V_2 are equal to those of the regions D_0 and D_2. Let us introduce the following assumption on the geometry of the region D:

$$R_1 \ll R_2 \ll R_0,\ l \ll R_0; \tag{1.1}$$

$$H_0 < \frac{1}{R_2},\ H_2 = 0\left(\frac{1}{R_2}\right). \tag{1.2}$$

Here H_0 and H_2 are the mean curvatures at the points of the surfaces Ω_0 and Ω_2 of the regions D_0 and D_2. Now suppose that the storage of external excitation is placed at the surface Ω_0 and the frequency of the excitation ω is close to one of the lowest free frequencies of acoustic vibrations ω_{0j} in the region D_0, i.e.,

$$\left|\frac{\omega - \omega_{0j}}{\omega_{0j}}\right| \ll 1. \tag{1.3}$$

Let us suppose that a gas jet issues from the resonator throat D_1 with velocity $U = \text{const}$ to enable active absorption of acoustic energy in the region D_0. We will simulate the jet by a cylinder with diameter $\rho = R_1$, and length $L = O(R_0)$. In this case

$$M = U/c \ll 1 \tag{1.4}$$

where c is the speed of sound of the gas. According to assumptions (1.1) and (1.3), we introduce the small parameters

$$\delta = R_1/R_2; \tag{1.5}$$

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$$\varepsilon = k = \omega R_2/c, \tag{1.6}$$

which will be used in constructing the approximate solution to the problem. We present the assumptions (1.3) and (1.4) as the estimates

$$k - k_{0j} = O(\varepsilon^{3/2}),\ k_{0j} = \omega_{0j} R_2/c; \tag{1.7}$$

$$M = O(\varepsilon). \tag{1.8}$$

2. Statement of the Problem and the Method of Solution. According to the assumptions introduced in Section 1, the problem of evaluating the acoustic vibrations in the region $D = D_0 \cup D_1 \cup D_2$ is reduced to finding the amplitude function of the velocity potential to accuracy up to the first order of smallness of ε. The function should satisfy the equation

$$\Delta\varphi + \lambda^2\varphi = 0,\ \lambda^2 = \omega^2/c^2 \tag{2.1}$$

and the following boundary conditions:
1) the impermeability condition

$$\frac{\partial\varphi}{\partial\nu} = 0,\ \mathbf{r} \in \Omega_1 \cup \Omega_2 \tag{2.2}$$

where ν is the direction of the internal normal to the rigid surface of the region D and r is the radius-vector of the gas particles;
2) the condition of dynamic compatibility at the jet boundary Γ, which is the surface of contact discontinuity of steady and unsteady components of the velocities

$$i\lambda\varphi^+ = i\lambda\varphi^- + M\frac{\partial\varphi^-}{\partial x},\ \rho = R_1,\ 0 \leqslant x \leqslant L \tag{2.3}$$

where φ^+ and φ^- are the values of φ at the outer and inner parts of Γ;
3) the Joukowskii–Kutta condition

$$|\nabla\varphi| < \infty,\ x = 0^+,\ \rho = R_1; \tag{2.4}$$

4) the condition of radiation of acoustic excitation energy by the surface Ω_0 of the region D_0:

$$\frac{\partial\varphi}{\partial\nu} = q(\mathbf{r}),\ \mathbf{r} \in \Omega_0. \tag{2.5}$$

Since the domain of solution of the problem stated is naturally divided into three simple subregions D_j (j = 0, 1, 2), it is reasonable to apply the matching method to find the solution. The method consists in construction of the sought-for function φ in each individual subregion separately and subsequent conjunction of the appropriate expressions for φ_j at the common parts of the boundaries D_j. In this case, taking account of the assumptions (1.1)-(1.4), the function φ_j can be found using the perturbation method. The elements of the method were employed in stating the problem in Eq. (2.1) wherein terms of the second order of smallness of ε were dropped. The latter circumstance considered, we will seek the solution of the problem approximately, namely, to accuracy up to the first order of smallness of ε and δ.

3. Solution in the Region D_1. Since the region D_1 is canonical and has the form of a circular cylinder, the general solution of Eq. (2.1) satisfying the condition (2.2) will be obtained in the axisymmetrical approximation using the method of separation of variables as follows:

$$\varphi_1 = \sum_{n=0}^{\infty}\left[a_n e^{\sqrt{\zeta_n^2 - \lambda^2}x} + b_n e^{-\sqrt{\zeta_n^2 - \lambda^2}(x + l)}\right] J_0(\zeta_n \rho). \tag{3.1}$$

Here $\zeta = 0$ and ζ_n (n = 1, 2,...) is the root of the equation

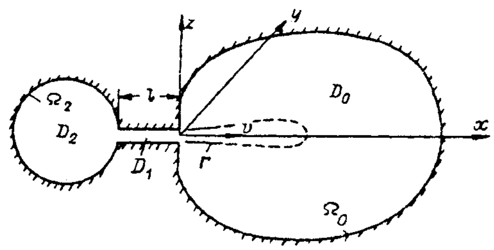

Fig. 1

$$\frac{\partial J_0}{\partial \rho} = 0 \text{ when } \rho = R_1;$$

J_0 is a Bessel function of the zeroth order.

4. Solution in the Region D_2. Acoustic vibrations in the region D_2 are excited by the acoustic energy flux from the region D_1. The function φ_2 describing the vibrations can be derived by solving Eq. (2.1) subject to (2.2) at the solid boundary of the region D_2 and the condition

$$\frac{\partial \varphi_2}{\partial x} = \frac{\partial \varphi_1}{\partial x} \text{ when } x = -l,\ 0 \leqslant \rho < R_1. \tag{4.1}$$

Let us present the function as the sum of three components

$$\varphi_2 = \varphi_{21} + \bar{\varphi}_2 + \psi_2, \tag{4.2}$$

where φ_{21} is the solution to Eq. (2.1) subject to the following boundary conditions:

$$\frac{\partial \varphi_{21}}{\partial x} = \begin{cases} \dfrac{\partial \varphi_1}{\partial x} \text{ when } x = -l,\ 0 \leqslant \rho \leqslant 1, \\ 0 \text{ when } x = -l,\ \rho > 1, \end{cases}$$
$$\lim_{|r| \to \infty} \left\{ r \left[\frac{\partial \varphi_{21}}{\partial r} + i\lambda \varphi_{21} \right] \right\} = 0; \tag{4.3}$$

where $\bar{\varphi}_2$ is an arbitrary function subject to the conditions

$$\frac{\partial \bar{\varphi}_2}{\partial \nu} = -\frac{\partial \varphi_{21}}{\partial \nu},\ \mathbf{r} \in \Omega_2; \tag{4.4}$$

$$\frac{\partial \bar{\varphi}_2}{\partial \nu} = 0,\ \mathbf{r} \in \Omega_{21}. \tag{4.5}$$

Here Ω_{21} is the common part of the boundaries of the regions D_2 and D_1, and ψ_2 is the solution to the equation

$$\Delta \psi_2 + \lambda^2 \psi_2 = -(\Delta \bar{\varphi}_2 + \lambda^2 \bar{\varphi}_2) = f \tag{4.6}$$

provided that

$$\frac{\partial \psi_2}{\partial \nu} = 0,\ \mathbf{r} \in \Omega_2 \cup \Omega_{21}. \tag{4.7}$$

The solution to the problem (4.3), which describe the acoustic field radiated by the section of the surface Ω_{21} of a flat screen, can be accurately presented using the Huygens–Rayleigh integral [2]

$$\varphi(\mathbf{r}) = -\frac{1}{2\pi} \int_{\Omega} \frac{e^{-i\lambda|\mathbf{r} - \mathbf{r}_0|} v_\nu(\mathbf{r}_0) d\sigma_0}{|\mathbf{r} - \mathbf{r}_0|},\ v_\nu = \frac{\partial \varphi}{\partial \nu}. \tag{4.8}$$

Passing to the dimensionless parameters

$$\bar{x} = -\frac{x+l}{R_1}, \bar{\rho} = \frac{\rho}{R_1}, \bar{l} = \frac{l}{R_1}, \bar{\zeta}_n = \zeta_n R_1, \bar{r} = \frac{r}{R_1}, \tag{4.9}$$

taking account of (1.5), (1.6), and (3.1), (4.3), and (4.9), we find from (4.8) for $\theta = 0$

$$\varphi_{21}(\bar{x}, \bar{\rho}) = \frac{R_1}{2\pi}\int_0^{2\pi}\int_0^1 \frac{\bar{\rho} v_{12} e^{-i\varepsilon\delta\Delta\bar{r}} d\bar{\rho}_0 d\theta_0}{\Delta\bar{r}}, \tag{4.10}$$

where, according to (4.1)

$$v_{12} = \frac{-\partial\varphi_1}{\partial x} = \sum_{n=0}^{\infty} A_n J_0(\bar{\zeta}_n\bar{\rho}), \ \bar{x} = 0; \tag{4.11}$$

$$\Delta\bar{r} = \sqrt{\bar{\rho}_0^2 - 2\bar{\rho}_0\bar{\rho}\cos\theta_0 + \bar{\rho}^2 + \bar{x}^2}; \tag{4.12}$$

$$A_n = \zeta_n(a_n e^{-\bar{\xi}_n \bar{l}} - b_n), \ A_0 = i\frac{\varepsilon}{R_2}(a_0 e^{-i\varepsilon\delta\bar{l}} - b_0). \tag{4.13}$$

By virtue of axial symmetry φ_1, the expression (4.10) for φ_{21} will be valid for any θ. Let us specify an arbitrary function $\tilde{\varphi}_2$ which must satisfy the conditions (4.4) and (4.5),

$$\tilde{\varphi}_2 = f_1(\nu) f_2(\mathrm{r}), \ \mathrm{r}(\xi, \eta) \in \Omega_2, \tag{4.14}$$

where

$$f_1 = \begin{cases} R_1 \int_{\bar{\nu}}^{1} \exp\left(-\frac{\bar{\nu}^2}{1-\bar{\nu}^2}\right) d\bar{\nu} & \text{when } \bar{\nu} = \nu/R_1 \leq 1, \\ 0 & \text{when } \bar{\nu} > 1; \end{cases} \tag{4.15}$$

$$f_2 = \frac{\partial\varphi_{21}}{\partial\nu} = \frac{\partial\varphi_{21}}{\partial r}\cos(\widehat{\mathrm{r}, \mathrm{v}}) \tag{4.16}$$

(ν, ζ, η) is the system of orthogonal curvilinear coordinates for which $\nu = 0$ is the equation of the surface Ω_2.

To determine the function ψ_2, let us expand the right-hand side of Eq. (4.6) into a series with respect to the eigenfunctions of the problem (4.6) and (4.7):

$$f = \sum_{n=0}^{\infty} d_n \psi_{2n}. \tag{4.17}$$

Then

$$\psi_2 = \sum_{n=0}^{\infty} \frac{d_n}{\lambda^2 - \lambda_{2n}^2} \psi_{2n}. \tag{4.18}$$

Here λ_{2n} are the eigenvalues of the problem (4.6) and (4.7)

$$\lambda_{20} = 0, \ \psi_{20} = 1. \tag{4.19}$$

when $n = 0$. Normalizing the eigenfunctions so that

$$\int_{V_2} \psi_{2n}^2 dv = V_2, \tag{4.20}$$

we find from (4.17)

$$d_n = \frac{1}{V_2} \int_{V_2} f \psi_{2n} dv. \tag{4.21}$$

Substituting the expression (4.6) for f in (4.21) with n = 0 and applying Green's formula, taking account of (4.19), we have

$$d_0 = -\frac{1}{V_2}\left(\lambda^2 \int_{V_2} \bar{\varphi}_2 dv + \int_{\Omega_2} \frac{\partial \bar{\varphi}_2}{\partial \nu} d\sigma\right). \tag{4.22}$$

Taking account of (4.3), (4.4), and (4.11), we obtain from (4.22)

$$d_0 = \pi A_0 \frac{R_1^2}{V_2}\left[1 + O(\delta\varepsilon^2)\right]. \tag{4.23}$$

Let us separate the zeroth term from (4.18) and estimate the remaining sum $\tilde{\psi}_2$ applying the Hölder inequality

$$|\tilde{\psi}| \leq \left(\sum_{n=1}^{\infty} d_n^2\right)^{1/2} \left(\sum_{n=1}^{\infty} \frac{\psi_{2n}^2}{(\lambda^2 - \lambda_{2n}^2)^2}\right)^{1/2}. \tag{4.24}$$

From the fullness of the system of eigenfunction, taking account of (4.20) it follows that

$$\sum_{n=0}^{\infty} d_n^2 = \frac{1}{V_2} \int_{V_2} f^2 dv. \tag{4.25}$$

Substituting (4.6) into (4.25) and taking into account (1.2), (4.10), (4.11), and (4.14)-(4.16), we obtain the estimate

$$\sum_{n=0}^{\infty} d_n^2 = \frac{R_2}{V_2} A_0^2 O(\delta^3). \tag{4.26}$$

Taking into account the asymptotic behavior of the eigenfunctions and eigenvalues [3] in the region V_2, we have

$$\sum_{n=1}^{\infty} \frac{\psi_{2n}^2}{(\lambda^2 - \lambda_{2n}^2)^2} = O(1). \tag{4.27}$$

Substituting (4.21) into (4.18) with taking into account (1.5), (1.6), and (4.23)-(4.27), we obtain the main component of (4.2) for φ_2:

$$\psi_2 = \frac{R_2^4 \delta^2}{V_2 k^2} [1 + \varepsilon^2 O(\delta^{-1/2})] A_0. \tag{4.28}$$

5. Solution in the region D_0. In the region D_0 the gas vibrates under the action of three sources with different excitation mechanisms. These sources are the jet issuing from the throat of the resonator and the acoustic energy flux from the resonator and from the external source. Accordingly, the amplitude function of the velocity potential in D_0 can be divided into three components:

$$\varphi_0 = \varphi_c + \varphi_{01} + \varphi_q. \tag{5.1}$$

To present the function φ_c, which describes the vibrations due to the interaction of the jet with the ambient environment, let us consider first the qualitative mechanism of the interaction. As is known [4], the steady component of the jet generates noise, whose level is proportional to the eighth power of the velocity of the gas flow in the jet. Taking into account (1.8), let us neglect the effect of the noise on the acoustic vibrations under consideration. Since the gas jet issuing from the resonator throat is affected by the external source of acoustic disturbances in the region D_0, the velocity of its flow will include also an unsteady component. Given the flow, the vibratory motion of the gas in the vicinity of the resonator edge is partially transformed into a vortex motion [5]. The appropriate eddying particles separating from the edge move with a velocity equal to that of the flow and form a vortex sheet of varying intensity. Under the assumption that during the motion of eddying intensity of certain particles of the sheet varies slightly [6], we will also ignore the acoustic disturbances which are generated by the unsteady jet component in the region D_0.

However, in addition to the acoustic disturbances the unsteady vortex sheet induces also a solenoidal component of the velocity variations of the gas. The value of the component in the vicinity of the resonator throat can affect significantly the acoustic energy flux from the resonator in the region D_0. Thus, as a basis of the function φ_c we take the amplitude function of the velocity potential φ_B, the velocity induced by the unsteady vortex sheet. The function satisfies the Laplace equation. Imposing on φ_B the condition of axial symmetry and the following condition to simplify the calculations:

$$\frac{\partial \varphi_B}{\partial x} = 0 \text{ when } x = 0, \tag{5.2}$$

let us present the function φ_c as the sum of two components:

$$\varphi_c = \varphi_B + \tilde{\varphi}_B. \tag{5.3}$$

Here $\tilde{\varphi}_B$ is the discrepancy due to inaccurate fulfillment of Eq. (2.1) and boundary condition (2.2) by the function φ_B.

The unsteady component of the intensity of the vortex sheet γ can be determined from (2.3). To this end, we differentiate the relationship (2.3) with respect to x and modify it as follows:

$$i\lambda\gamma + \frac{1}{2}M\frac{\partial \gamma}{\partial x} = M\frac{\partial v_x}{\partial x},$$

where $\gamma = \frac{\partial \varphi^-}{\partial x} - \frac{\partial \varphi^+}{\partial x}$, $v_x = \frac{1}{2}\left(\frac{\partial \varphi^-}{\partial x} + \frac{\partial \varphi^+}{\partial x}\right)$. Let us write the solution to the equation in the dimensionless coordinates

$$\gamma = \gamma_0 \exp(-i\alpha\bar{x}) + \gamma_1,\ \alpha = 2\omega R_1/U. \tag{5.4}$$

Here γ_0 is the intensity of the vortices shedding from the edge of the resonator, which is found from the condition (2.4):

$$\gamma_0 = \frac{1}{R}\frac{\partial \varphi_1}{\partial \bar{x}} = v_0 \text{ when } x \to 0^-,\ \bar{\rho} = 1; \tag{5.5}$$

γ_1 is the component of γ which appears due to the tension and compression of the jet in the acoustic velocity field:

$$\gamma_1 = -2e^{-i\alpha\bar{x}}\int_0^{\bar{x}} \frac{\partial v_x}{\partial \bar{x}} e^{i\alpha\bar{x}} d\bar{x} = -2v_x[1 + O(\delta)].$$

By virtue of (1.3) the acoustic vibrations in the region D_0 occur with predominance of the j-th harmonic. Therefore, the function v_x can be approximately presented as follows:

$$v_x = g_j\frac{1}{R_0}\sin\beta\bar{x},\ \beta = O(\varepsilon\delta)$$

(g_j is the amplitude of ψ-th harmonic of acoustic vibrations in D_0). This yields

$$\gamma_1 = -2g_j\frac{1}{R_0}\sin\beta\bar{x}. \tag{5.6}$$

Taking into account the condition (5.2), let us determine the velocity field induced by the unsteady vortex sheet on the right half plane from the formula

$$\mathbf{v}_B = \nabla\varphi_B(\bar{r}) = -\frac{R_1}{4\pi}\int_{-L}^{L}\int_{-\pi}^{\pi}\frac{[(\mathbf{r} - \mathbf{r}_0) \times \tau]\bar{x}_0\gamma d\theta_0 d\bar{x}_0}{|\bar{x}_0|\,|\mathbf{r} - \mathbf{r}_0|^3},$$

where r is the dimensionless radius-vector related to R_1 and $\vec{\tau}_0$ is the unit-vector of the tangential to the surface of the vortex sheet, which has the same sense of direction as the vector of the elementary vortex that is placed at the point r_0.

Using the formula, we find the value of the function φ_B with $x = 0$ and $\rho < 1$. Taking into account the assumption of axial symmetry, we will assume that $\theta = 0$. It is shown below that the quantity φ_B, which is determined by the component

γ_1 of the vortex layer, is the value of the second order of smallness with respect to the excitation from the external source. Hence, substituting the first term (5.4) with account of (5.5) in the formula for v_B and taking into account that $L = O[(\varepsilon\delta)^{-1}]$, we obtain for the axial and radial components of the velocities

$$\frac{\partial\varphi_B}{\partial\bar{x}} = \frac{v_0R_1}{4\pi}\int_{-\infty}^{\infty}\int_{-\pi}^{\pi}\frac{(1-\bar{\rho}\cos\theta_0)\bar{x}_0e^{-i\alpha|\bar{x}_0|}d\theta_0d\bar{x}_0}{|\bar{x}_0|[1-2\bar{\rho}\cos\theta_0+\bar{\rho}^2+(\bar{x}-\bar{x}_0)^2]^{3/2}}; \tag{5.7}$$

$$\frac{\partial\varphi_B}{\partial\bar{\rho}} = \frac{v_0R_1}{4\pi}\int_{-\infty}^{\infty}\int_{-\pi}^{\pi}\frac{(\bar{x}-\bar{x}_0)\bar{x}_0e^{-i\alpha|\bar{x}_0|}d\theta_0d\bar{x}_0}{|\bar{x}_0|[1-2\bar{\rho}\cos\theta_0+\bar{\rho}^2+(\bar{x}-\bar{x}_0)^2]^{3/2}}. \tag{5.8}$$

From (5.8) it follows that

$$\frac{\partial\varphi_B}{\partial\bar{\rho}} = 0 \text{ when } \bar{\rho} = 0. \tag{5.9}$$

Taking into account (5.9), we find from the equations of gas motion that with $\bar{\rho} = 0$

$$\frac{\partial p_B}{\partial\bar{x}} = -\rho_0\frac{U}{R_1}e^{-i\alpha\bar{x}}\frac{\partial}{\partial\bar{x}}\left(e^{i\alpha\bar{x}}\frac{\partial\varphi_B}{\partial\bar{x}}\right), \tag{5.10}$$

where p_B is the amplitude function of the pressure fluctuations induced by the vortex sheet and ρ_0 is the gas density.

Integrating Eq. (5.10) and assuming that $p_B = 0$. With $\bar{x} \to \infty$, we obtain

$$p_B = \rho_0\frac{U}{R_1}\left[\frac{\partial\varphi_B}{\partial\bar{x}}\Big|_0^{\infty} + i\alpha\int_0^{\infty}\frac{\partial\varphi_B}{\partial\bar{x}}d\bar{x}\right] \text{ when } \bar{x} = \bar{\rho} = 0. \tag{5.11}$$

Substituting (5.7) into (5.11), we have

$$p_B(0) = \rho_0v_0UI_1. \tag{5.12}$$

Here

$$I_1 = \int_0^{\infty}\frac{e^{-i\alpha t}}{(1+t^2)^{3/2}}dt. \tag{5.13}$$

Substituting (5.12) into the Cauchy–Lagrange integral and taking account of (5.2), we obtain

$$\varphi_{B0} = i\frac{v_0U}{\omega}I_1 \text{ when } \bar{x} = \bar{\rho} = 0. \tag{5.14}$$

Taking into account (5.8), the function $\varphi_B(\rho)$ with $x = 0$ can be calculated by integrating the expression (5.8) with account of (5.14):

$$\varphi_B = \varphi_{B0} - 2v_0R_1I_2$$
$$\left(I_2 = \frac{1}{2\pi}\int_0^{\rho}\int_0^{\infty}\int_0^{\pi}F(\rho,\theta,t)d\theta dt d\rho,\ F(\rho,\theta,t) = \frac{te^{-i\alpha t}}{[1-2\rho\cos\theta+\rho^2+t^2]^{3/2}}\right). \tag{5.15}$$

Substituting (5.14) into (5.15), we obtain

$$\varphi_B(\rho) = i\frac{v_0U}{\omega}I\ (I = I_1 + i\alpha I_2). \tag{5.16}$$

Now let us estimate the component φ_{B1}, which is determined by the value of the component γ_1. Substituting (5.6) into the formula for $\bar{v}_B$, comparing the appropriate projections with (5.7) and (5.8), and taking account of (5.113) and (5.14), we have

$$\varphi_{B1} = \frac{g_jU}{\omega R_0}\mathrm{Im}\int_0^{\infty}\frac{e^{-i\beta t}}{(1+t^2)^{3/2}}dt = g_jO(\varepsilon^2\delta). \tag{5.17}$$

The function $\tilde{\varphi}_B$ (5.3) can be determined by solving the equation

$$\Delta\tilde{\varphi}_B + \lambda^2\tilde{\varphi}_B = -\lambda^2\overline{\varphi}_B \tag{5.18}$$

with boundary condition

$$\frac{\partial\tilde{\varphi}_B}{\partial\nu} = -\frac{\partial\varphi_B}{\partial\nu},\ \mathrm{r} \in \Omega_0 \cup \Omega_{01}. \tag{5.19}$$

Let us present the amplitude function of the velocity potential φ_{10} (5.1), which describes the acoustic vibrations in the region D_0, the vibrations being excited by the acoustic energy flux from the region D_1, as follows:

$$\varphi_{01} = \overline{\varphi}_{01} + \tilde{\varphi}_{01}, \tag{5.20}$$

where $\bar{\varphi}_{01}$ is the main part of the function φ_{01}, which we will determine as the solution to Eq. (2.1) subject to the following boundary conditions:

$$\frac{\partial\overline{\varphi}_{01}}{\partial x} = \begin{cases} \dfrac{\partial\varphi_1}{\partial\overline{x}} \text{ when } \overline{x} = 0,\ 0 \leqslant \overline{\rho} \leqslant 1, \\ 0 \text{ when } \overline{x} = 0,\ \overline{\rho} > 1; \end{cases} \tag{5.21}$$

$$\overline{\varphi}_{01} = 0 \text{ when } \overline{x} \to \infty. \tag{5.22}$$

The solution to the problem (5.21) and (5.22), analogous to the solution (4.10) of the problem (4.3), has the form

$$\overline{\varphi}_{01}(\overline{x}, \overline{\rho}) = \frac{R_1}{2\pi}\int_0^{2\pi}\int_0^1 \frac{\overline{\rho}_0 v_{10} e^{-i\varepsilon\delta\Delta\overline{r}}}{\Delta\overline{r}}\, d\overline{\rho}_0 d\theta_0. \tag{5.23}$$

Here

$$v_{10} = \frac{\partial\varphi_1}{\partial\overline{x}} = \sum_{n=0}^{\infty} B_n J_0(\zeta_n\overline{\rho}),\ \overline{x} = 0; \tag{5.24}$$

$$B_0 = i\frac{\varepsilon}{R_2}\left(b_0 e^{-i\varepsilon\delta\overline{l}} - a_0\right); \tag{5.25}$$

$$B_n = \zeta_n\left(b_n e^{-\zeta_n\overline{l}} - a_n\right). \tag{5.26}$$

The function $\tilde{\varphi}_{01}$ of Eq. (5.20) can be determined by solving Eq. (2.1) with the boundary condition

$$\frac{\partial\tilde{\varphi}_{01}}{\partial\nu} = \begin{cases} -\dfrac{\partial\overline{\varphi}_{01}}{\partial\nu}, & \mathrm{r} \in \Omega_0, \\ 0, & \mathrm{r} \in \Omega_{01}. \end{cases} \tag{5.27}$$

Finally, we will determine the function φ_q of Eq. (5.1) as the solution of Eq. (2.1) subject to (2.5) and the condition

$$\frac{\partial\varphi_q}{\partial x} = 0,\ \mathrm{r} \in \Omega_{01}. \tag{5.28}$$

Let us introduce the function

$$\tilde{\varphi}_0 = \tilde{\varphi}_B + \tilde{\varphi}_{01} + \varphi_q, \tag{5.29}$$

whose components should be the solutions of the problems (5.18) and (5.19); (2.1) and (5.27); and (2.1) and, (2.5), and (5.28), and present it as the sum of two components:

$$\tilde{\varphi}_0 = \tilde{\psi}_0 + \psi_0$$

where $\tilde{\psi}_0$ is an arbitrary function which satisfies the boundary conditions (2.5), (5.19), (5.27), and (5.28) in the aggregate. Then the definition of the function ψ_0 reduces to the solution of the equation

$$\Delta\psi_0 + \varepsilon^2\delta^2\psi_0 = f_0,\ f_0 = -(\Delta\tilde{\psi}_0 + \varepsilon^2\delta^2\tilde{\psi}_0) \tag{5.30}$$

with uniform Neumann conditions at the boundary $\bar{\Omega}_0 = \Omega_0 \cup \Omega_{01}$. Let us present the function $\tilde{\psi}_0$, as well as the function φ_2, in the form (4.14)-(4.16). Having presented the function ψ_0 as a series in terms of the eigenfunctions of the problem (5.30), let us find $\tilde{\psi}_0$ in the same manner as ψ_2 in Section 4:

$$\tilde{\psi}_0 = -\frac{R_2^4}{V_0}\frac{\delta^2}{k^2 - k_{0j}^2}\{I_{qj}[\psi_{0j} + O(\varepsilon^2)] + \pi B_0\psi_{0j}^*[\psi_{0j} + O(\varepsilon^3\delta^{-1/2})]\}, \tag{5.31}$$

where ψ_{0j}^* is the value of the eigenfunction ψ_{0j} with $\bar{x} = \bar{\rho} = 0$ and B_0 is the constant (5.25),

$$I_{qj} = \int_{\Omega_0} \psi_{0j} q\, d\bar{\sigma}. \tag{5.32}$$

6. Matching of Solutions. The functions φ_j (j = 0, 1, 2), presented above in the appropriate subregions D_j, will determine the general solution of the problem stated in the matching conditions hold:

$$\varphi_0 = \varphi_1,\ \mathbf{r} \in \Omega_{10}; \tag{6.1}$$

$$\varphi = \varphi_2,\ \mathbf{r} \in \Omega_{12}. \tag{6.2}$$

Substituting the main parts of the asymptotic presentations of the function φ_j found in Sections 3-5 into the conditions (6.1) and (6.2), we obtain

$$a_0 + b_0 e^{-i\lambda l} + \sum_{n=1}^{\infty} a_n J_0(\zeta_n\bar{\rho}) = i\frac{v_0 U}{\omega} I + \frac{R_1}{2\pi}\int_0^{2\pi}\int_0^1 \frac{\bar{\rho}_0 v_{10}}{\Delta\bar{r}}\, d\bar{\rho}_0 d\theta_0 - \frac{\delta^2 R_2^4}{V_0(k^2 - k_{0j}^2)}(I_{qj} + \pi B_0\psi_{0j}^*\psi_{0j}); \tag{6.3}$$

$$a_0 e^{-i\lambda l} + b_0 + \sum_{n=1}^{\infty} b_n J_0(\zeta_n\bar{\rho}) = -\frac{R_1^2\pi}{\lambda^2 V_2}A_0 + \frac{R_1}{2\pi}\int_0^{2\pi}\int_0^1 \frac{\bar{\rho}_0 v_{12}}{\Delta\bar{r}}\, d\bar{\rho}_0 d\theta_0. \tag{6.4}$$

Expanding the right-hand sides of the relationships (6.3) and (6.4) at the section $(0 \le \bar{\rho} \le 1)$ into a Fourier–Bessel series of the second type and equating the coefficients of the series to the corresponding coefficients of the left-hand sides of the relationships, we obtain the following system of algebraic equations:

$$a_0 + b_0 e^{-i\lambda l} = R_2 v_0\left(i\frac{M}{k}I_1 - 2\delta\bar{I}_2\right) - \frac{\delta^2 R_2^4}{V_0(k^2 - k_{0j}^2)}(I_{qj} + \pi B_0\psi_{0j}^*)\psi_{0j}^* + Q_{10}; \tag{6.5}$$

$$a_0 e^{-i\lambda l} + b_0 = -\frac{\delta^2\pi R_2^4}{k^2 V_2}A_0 + Q_{12}; \tag{6.6}$$

$$a_n = \sum_{m=0}^{\infty} h_{nm}B_m + d_n v_0,\ n = 1, 2, \dots; \tag{6.7}$$

$$b_n = \sum_{m=0}^{\infty} h_{nm}A_m,\ n = 1, 2, \dots \tag{6.8}$$

Here

$$h_{nm} = \frac{R_1}{2\pi C_n}\int_0^{\rho}\int_0^{\infty}\int_0^{\pi} \frac{\bar{\rho}_0\bar{\rho}J_0(\zeta_n\bar{\rho})J_0(\zeta_m\bar{\rho}_0)}{\Delta\bar{r}}\, d\theta_0 d\bar{\rho} d\bar{\rho}_0;\ C_n = \int_0^1 \bar{\rho}J_0^2(\zeta_n\bar{\rho})d\bar{\rho};$$

$$d_n = \frac{R_1}{\pi\zeta_n C_n}\int_0^1\int_0^{\infty}\int_0^{\pi} \rho J_1 F(\rho, \theta, t)d\theta dt d\rho;\ \bar{I}_2 = \frac{1}{2\pi}\int_0^1\int_0^{\infty}\int_0^{\pi}(I - \rho^2)F(\rho, \theta, t)d\theta dt d\rho; \tag{6.9}$$

$$Q_{ij} = \frac{R_1}{\pi}\int_0^1\int_0^1\int_0^{2\pi} \frac{\bar{\rho}_0\bar{\rho}v_{1j}}{\Delta\bar{r}}\, d\theta_0 d\bar{\rho}_0 d\bar{\rho}\ (j = 0, 2);$$

A_m and B_m are expressed in terms of a_0 and b_0 by the formulas (4.12), (4.13), (5.25), and (5.26). The system (6.5)-(6.8) is closed by the relationship that follows from the condition (2.4). It follows from the expression (5.8) and (5.23) that the radial components of velocity at the edge of the resonator with $\bar{x} = c$ and $\bar{\rho} = 1$ have regularities of the type

$$\frac{\partial \varphi_B}{\partial \bar{\rho}} = -\frac{v_0 R_1}{2\pi} \int_0^{\pi} \frac{d\theta}{\sin\frac{\theta}{2}}, \quad \frac{\partial \bar{\varphi}_{01}}{\partial \bar{\rho}} = \frac{B_0 R_1}{2\pi} \int_0^{\pi} \frac{d\theta}{\sin\frac{\theta}{2}}.$$

Since

$$\left|\frac{\partial \bar{\varphi}}{\partial x}\right| < \infty, \quad \left|\frac{\partial \tilde{\varphi}}{\partial \rho}\right| < \infty,$$

the condition (2.4) will be valid if

$$v_0 = B_0 = i\frac{\varepsilon}{R_2}(b_0 e^{-i\varepsilon\delta\bar{l}} - a_0). \tag{6.10}$$

An asymptotic analysis of the coefficients h_{nm} with m and $n \to \infty$ shows that the system (6.7) and (6.8) reduces to the type of systems which appear in the history of waveguides [6] in solving the appropriate problems by the matching method. Taking this into account, as well as the expressions for A_m and B_m, we obtain the estimate

$$\| X \|_\infty = O(\varepsilon\delta) \| X_0 \|_\infty, \tag{6.11}$$

where X_0 and X are the vectors

$$X_0 = X_0\{a_0, b_0\}, \; X = X\{a_1, b_1, a_2, b_2, \dots\}.$$

Substituting Eqs. (4.11) and (5.24) for v_{ij} into formula (6.9) with account of (6.11), we come to the conclusion that the values of Q_{ij} in Eqs. (6.5) and (6.6) can be neglected. Thus, in the asymptotic approximation, Eqs. (6.5) and (6.6) with account of (6.9) will make up the system of closed equations with respect to the constants a_0 and b_0.

7. Dependence of the Level of Acoustic Vibrations in the Region D_0 on the Parameters of Resonator and Jet. Taking account of (5.1), (5.3), (5.20), and (5.29), we modify the expression for the amplitude function of gas vibrations in the region D_0 as follows:

$$\varphi_0 = \varphi_B + \bar{\varphi}_{01} + \tilde{\varphi}_0$$

where φ_B is the solenoidal component, and $\bar{\varphi}_{01}$ and $\tilde{\varphi}_0$ are the acoustic components. In this case the function $\bar{\varphi}_{01}$ describing the acoustic gas radiation from the resonator with velocity v_0 rapidly vanishes as the distance from the resonator throat increases, while the function $\tilde{\varphi}_0$, which determines the resonance vibrations in the region D_0, is practically uniform over the whole region.

Solving the system (6.5) and (6.6) in terms of (6.11) we find from (6.10)

$$v_0 = \frac{k^2 V_2 \psi_{0j}^* I_{qj}}{\pi V_0\left[\left(I - \frac{k^2}{k_{0j}^2}\right) H - \mu\right] k_{0j}^2},$$

and from (5.31)

$$\tilde{\varphi}_0 = i\frac{\bar{k}^2}{k^2} H \frac{\psi_{0j}}{\psi_{0j}^*} v_0,$$

where

$$H = \left(I - \frac{k^2}{\tilde{k}^2}\right) - \iota \overline{M}; \ \tilde{k}^2 = \frac{S_1 R_2^2}{V_2 l}; \ \overline{M} = \frac{R_2}{l} \frac{Mk}{\tilde{k}^2} I_1; \ \mu = \frac{V_2}{V_0} \frac{k^2}{k_{0j}^2} \psi_{0j}^{*2};$$

S_1 is the cross-sectional area of D_1 and $\tilde{k}$ is the reduced frequency of the resonator.

Let us analyze the dependence of the amplitude–frequency response of acoustic vibrations in D_0 on the resonator parameters using the magnification function

$$\eta(k) = \left| \frac{\bar{\varphi}_0(k)}{\bar{\varphi}_0(0)} \right| = \frac{1}{\left(I - \frac{k^2}{\tilde{k}^2}\right) - \frac{\mu}{H}}. \tag{7.1}$$

In the absence of the jet ($M = 0$) with $k = \tilde{k}$ we have $\eta = 0$. In this case, as is known, reactive suppression of the vibrations takes place. Along with this, $M = 0$, when the resonator is well-tuned to the frequency ω_{0j} in D_0, i.e., according to (1.7),

$$\tilde{k} \approx k_{0j}(I + \tilde{r}), \ \tilde{r} = O(\varepsilon^{1/2}), \tag{7.2}$$

in a certain small vicinity k_{0j}, namely with

$$k = k_{0j}(I + r_p), \ r_p = \frac{1}{2}\left(\tilde{r} \pm \sqrt{\tilde{r}^2 + \mu}\right), \ \mu = O(\varepsilon) \tag{7.3}$$

the quantity η becomes infinity, i.e., with (7.3), resonance in the whole region D takes place. This is a substantial shortcoming of the resonator as a reactive absorber.

Given the jet ($M \neq 0$), the denominator of (7.1) is a complex value; therefore, the quantity η cannot become infinity. Physically, in this case a portion of the acoustic energy transforms into vortex energy, which results in its active absorption. Let us consider the quantity η with $M \neq 0$ in the regime of resonance of the system, assuming approximately that the equality to zero of the real part of the denominator (7.1) is the condition governing the resonance:

$$2r_p[(2\tilde{r} - 2r_p + \overline{M}'')^2 + \overline{M}'^2] + \mu(2\tilde{r} - 2r_p + \overline{M}'') = 0. \tag{7.4}$$

Here $\tilde{r}$ is the parameter of detuning of the frequency of the resonator and the free frequency ω_{0j} (7.3); $\bar{M}'$ and $\bar{M}''$ are the real and imaginary parts of $\bar{M}$, and r_p is the correction of the eigenvalue k_{0j} for the region D_0 for the attachment of the active resonator (regions D_1 and D_2) which is the solution to Eq. (7.4).

With the constraint (7.4) the expression (7.1) can be modified to give

$$\eta(k_p) = \frac{(I - \mu)|(2\tilde{r} - 2r_p + \overline{M}'')^2 + \overline{M}'^2|}{\mu \overline{M}'} = \frac{(I - \mu)|2\tilde{r} - 2r_p + \overline{M}''|}{2\overline{M}'|r_p|}. \tag{7.5}$$

Equations (7.4) and (7.5) in the aggregate determine the dependence of the value $\eta(k_p)$ on the parameters of the resonator μ, r, and $\bar{M}$. In this case the parameter $\bar{M}$, which depends on the Mach number of the jet, serves as the damping factor of the system under consideration. As is known from the theory of active absorbers [7], the dependence of the level of the resonance vibrations on the damping factor should be nonmonotonic. Therefore, of practical interest is the assessment of the optimum value $M = M_0$ with which the maximum absorption of the resonance vibration occurs. This can be found from Eq. (7.4) and the condition

$$\frac{\partial \eta(k_p)}{\partial M} = 0. \tag{7.6}$$

In an explicit form, we failed to obtain the solution to the system (7.4) and (7.6) with respect to M_0; thus, we present only the estimate

$$M_0 = O(\varepsilon^{3/2}),$$

$$\eta(k_p) = O(\varepsilon^{-1/2}),$$

which follows from (7.5) with regard for (1.6), (5.13), (7.2), and (7.3), and with $\eta(k_p) = O(\varepsilon^{-1/2})$, while with $M \to 0$, $\eta(k_p) \to \infty$; and with $M = O(\varepsilon)$, $\eta(k_p) \to O(\varepsilon^{-1})$.

REFERENCES

1. V. B. Kurzin, "Active resonator as a dynamic absorber of acoustic vibration in a bounded volume," Prikl. Mekh. Tekh. Fiz., No. 2 (1991).
2. E. Skudrzyk, Fundamentals of Acoustics, Vol. 2, Springer-Verlag, Wien–New York (1971), V. 2.
3. R. Courant and D. Hilbert, Methods of Mathematical Physics, Berlin (1931).
4. M. E. Goldstein, Aeroacoustics, McGraw-Hill, New York (1976).
5. J. H. M. Disseihorst and L. Van Wijngaarden, "Flow in the exit of open pipes during acoustic resonance," J. Fluid Mech., 99, 2 (1980).
6. D. I. Blokhintsev, Acoustics of a Nonhomogeneous Moving Medium, Nauka, Moscow (1981).
7. J. P. Den Hartog, Mechanical Vibrations, 4th edn., McGraw-Hill, New York (1956).